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usual formulas, $FP=\frac{a^2-ex_1}{a}$; and the lengths of the perpendicular from the point $\left(\frac{x_1+e}{2}, \frac{y_1}{2}\right)$ upon CQ is $\frac{y_1-(x_1+e)\tan\theta}{\pm 2\sqrt{1+\tan\theta}}$.

Since CQ is tangent to the circle upon FP this perpendicular$=\frac{FP}{2}$.

Therefore, substituting for (x_1, y_1) their values in terms of θ, we obtain,

$$(a-b)\sin\theta\cos\theta-a=-e(\sin\theta+\cos\theta).$$

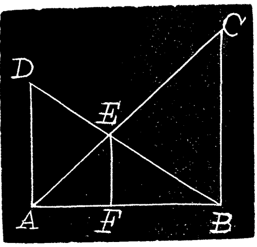
Squaring and arranging $(a-b)^2\sin^2\theta\cos^2\theta-2(a-b)(2a+b)\sin\theta\cos\theta=-b^2$, which may be written $(a-b)^2\sin^2 2\theta-4(a-b)(2a+b)\sin 2\theta=-4b^2$. Solving, $\sin 2\theta=\frac{2(2a+b)\pm4\sqrt{[a(a+b)]}}{a-b}$.

Excellent solutions were also received from *J. F. W. SCHEFFER*, *G. B. M. ZERR*, and *JOHN FAUGHT*.

24. Proposed by T. W. PALMER, Professor of Mathematics in the University of Alabama.

Two right triangles have the same base, the hypotenuses of the first is equal to 60, of the second 40. The point of intersection of the two hypotenuses is at the distance 15 from the base. Find the length of the base.

Solution by SETH PRATT, C. E., Assyria, Michigan; A. H. BELL, Hillsboro, Illinois; and P. S. BERG, Apple Creek, Ohio.



Let $AD=x-y$; $BC=x+y$; $EF=c=15$; $BD=b=40$; $AC=a=60$; $AF=z$; and $FB=w$. By similar triangles, $w:c=w+z:x-y$, or $w(x-y)=c(w+z)$....(1); and $z:c=w+z:x+y$, or $(w+z)c=z(x+y)$....(2).

$\therefore w(x-y)=z(x+y)$ ...(3). Put $x+y=s$ and $x-y=t$. Then $wt=zs$ or $w=zs/t$....(4). Substituting this value of w in the proportion of (2), $\frac{2s}{t}:c=\frac{zt+zs}{t}:t$. Dividing antecedents by $\frac{z}{t}$, we have $s:c=t+s:t$, or $st=c(t+s)$, whence $c=\frac{st}{s+t}$*$=\frac{(x+y)(x-y)}{x+y+x-y}=\frac{x^2-y^2}{2x}$....(5). From (5), $x^2-2cx=y^2$, whence $x=c+\sqrt{(y^2+c^2)}$. From the figure, $AC^2-BC^2=AB^2$ and $BD^2-AD^2=AB^2$. $\therefore AC^2-BC^2=BD^2-AD^2$, or $a^2-(x+y)^2=b^2-(x-y)^2$, whence $x=\frac{a^2-b^2}{4y}$. Equating the two values of x, $\frac{a^2-b^2}{4y}=c+\sqrt{(y^2+c^2)}$.

From this, we have $y^4+\frac{1}{2}c(a^2-b^2)y=(a^2-b^2)^2$.

Restoring numbers, and solving the resulting equation, $y=14.060811$.

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*The sequence is, that *AB* may be any number whatever when *AC* and *BD* are indefinite.

$$x=\frac{a^2-b^2}{y}=\frac{500}{y}=35.559825.\quad x+y=49.620636,\ x-y=21.499014.$$

$$AB=\sqrt{(AC^2-BC^2)}=\sqrt{[a^2-(x+y)^2]}=33.731178.$$

Solutions were also received from *G. B. M. ZERR*, *J. F. W. SCHEFFER*, *L. B. FRAKER*, and *J. K. ELLWOOD.*

25. Proposed by L. B. FRAKER, Weston, Ohio.

The sides of a quadrilateral board are $AB=7$ inches, $BC=15$ inches, $CD=21$ inches, and $DA=13$ inches; radius of incribed circle is 6 inches. (1) What are dimensions of the largest rectangular board that can be cut out of the given board, (2) largest square, (3) largest equilateral triangle? (Please solve without use of the calculus.)

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $Aa=Ad=x$, $Ba=Bb=y$, $Cb=Cc=z$, $Dc=Dd=w$,

$$\text{then } x+y=AB=7=6\cot\tfrac{1}{2}A+6\cot\tfrac{1}{2}B\ldots\ldots(1),$$

$$y+z=BC=15=6\cot\tfrac{1}{2}B+6\cot\tfrac{1}{2}C\ldots\ldots(2),$$

$$z+w=CD=21=6\cot\tfrac{1}{2}C+6\cot\tfrac{1}{2}D\ldots\ldots(3),$$

$$w+x=DA=13=6\cot\tfrac{1}{2}D+6\cot\tfrac{1}{2}A\ldots\ldots(4).$$

$$\therefore\ 28=6\cot\tfrac{1}{2}A+6\cot\tfrac{1}{2}B+6\cot\tfrac{1}{2}C+6\cot\tfrac{1}{2}D\ldots\ldots(5).$$

$(5)-\{(1)+(4)\}$ gives $4=3\cot\frac{1}{2}C-3\cot\frac{1}{2}A$

$$\therefore\ 4\sin\tfrac{1}{2}A\sin\tfrac{1}{2}C=3\sin\tfrac{1}{2}A\cos\tfrac{1}{2}C-3\cos\tfrac{1}{2}A\sin\tfrac{1}{2}C\ldots\ldots(6).$$

$(5)-\{(3)+(4)\}$ gives $1=\cos\frac{1}{2}D-\cos\frac{1}{2}B$

$$\therefore\ \sin\tfrac{1}{2}D\sin\tfrac{1}{2}B=\sin\tfrac{1}{2}B\cos\tfrac{1}{2}D-\cos\tfrac{1}{2}B\sin\tfrac{1}{2}D\ldots\ldots(7).$$

From DB^2 we get $45\cos C-13\cos A=32.$

$$\therefore\ 13\sin^2\tfrac{1}{2}A=45\sin^2\tfrac{1}{2}C\ldots\ldots(8).$$

$$45\cos^2\tfrac{1}{2}C-13\cos^2\tfrac{1}{2}A=32\ldots\ldots(9).$$

From AC^2 we get $13\cos D-5\cos B=8.$

$$\therefore\ 13\sin^2\tfrac{1}{2}D=5\sin^2\tfrac{1}{2}B\ldots\ldots(10).$$

$$13\cos^2\tfrac{1}{2}D-5\cos^2\tfrac{1}{2}B=8\ldots\ldots(11).$$

From (6), (7), (8), (9), (10), (11) we get

$$A=134^\circ\ 45'\ 35'',\ B=106^\circ\ 15'\ 39'',\ C=D=59^\circ\ 29'\ 23''.$$

Now the greatest equilateral triangle is the one which has the greatest side. Since $D=C$ only two cases confront us, the one is to draw a line from B making an angle of 60° with DC, the other to draw a line from some point in DC making an angle of 6° with BC. The former line is found to be very little the longer. Let BL be this line, then we have $15:BL=\sin 120^\circ:\sin 59^\circ\ 29'\ 23''$.

$\therefore$ $BL=$side of triangle$=14.92$ inches. $\therefore$ $BLM=$triangle. Also since $D=C$, the greatest square has its side coincident with DC. Hence $\frac{21-x}{2}:x=\cos 59^\circ\ 29'\ 23'':\sin 59^\circ\ 29'\ 23''$.

$\therefore$ $x=$side of square$=9.64$ inches, and $PQRS=$square. Also since $D=C$, the side of the greatest rectangle will